

## Determination of viscosity with Ostwald viscometer

### Theory

#### Transport processes

In a transport process, some quantity is transported from one place to another.

The general transport equation:

$$J_i = L_i \cdot X_i$$

where  $J_i$  is the flux of the  $i^{\text{th}}$  species (the transported extensive quantity per unit area and time),  $L_i$  the conductivity coefficient,  $X_i$  the thermodynamic force (it is a gradient of an intensive quantity)

Types of the transport processes:

name	driving force	transported quantity	conductivity coefficient	describing law
<b>viscous flow</b>	velocity gradient $\frac{dv_y}{dx}$	impulse $\frac{F}{A} = \frac{1}{A} \frac{dmv_y}{dt}$	dynamic viscosity $\eta$	Newton's law $\frac{F}{A} = -\eta \frac{dv_y}{dx}$
<b>diffusion</b>	concentration gradient $\frac{dc}{dx}$	chemical species $\frac{1}{A} \frac{dn}{dt}$	diffusion coefficient $D$	Fick's 1 <sup>st</sup> law $\frac{1}{A} \frac{dn}{dt} = -D \frac{dc}{dx}$
<b>heat conductivity</b>	temperature gradient $\frac{dT}{dx}$	heat $\frac{1}{A} \frac{dq}{dt}$	heat conductivity $\lambda$	Fourier's law $\frac{1}{A} \frac{dq}{dt} = -\lambda \frac{dT}{dx}$
<b>electric conductivity</b>	potential gradient $\frac{d\Phi}{dx}$	charge $\frac{1}{A} \frac{dQ}{dt}$	specific conductivity $\kappa$	Ohm's law $\frac{1}{A} \frac{dQ}{dt} = -\kappa \frac{d\Phi}{dx}$

#### The Viscous Flow

Let us consider two plates having the same area, laying parallel to each other: let the space between them be filled with a fluid (either gas or liquid). If the upper plate is dragged by a force  $F$  in the  $y$  direction, then it shall move with a constant  $v_y$  velocity.

So the plate will move with a constant velocity, having zero acceleration: the sum of the forces that act on the plate is zero. Thus, against the force  $F$  that is dragging the plate, a frictional force, having the same magnitude but the opposite direction as  $F$  is acting.

It's an empirical fact, that in this case, the fluid layer near the plate is not moving as compared to the plate; so this frictional force does not arise on the solid-fluid interface, but between the fluid layers: the adjacent layers are moving with different velocities.

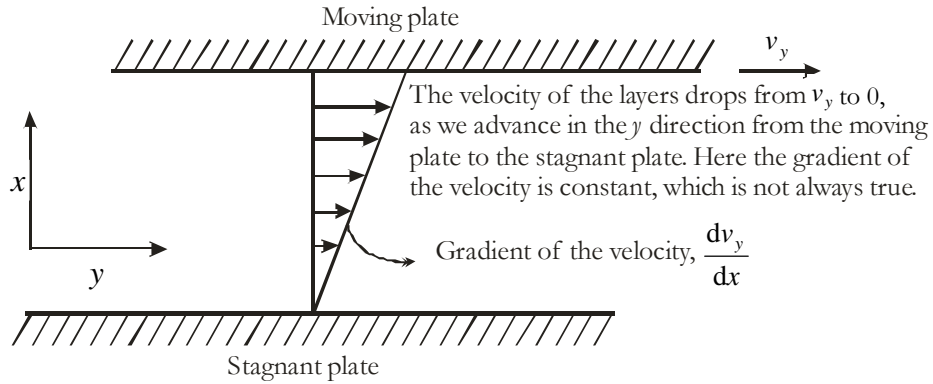


Fig. 1 For the explanation of viscous flow.

So when dealing with viscous flow every fluid layers chafe on their neighbours. For two fluid layers, interacting with each other through a surface  $A$ , laying at an infinitesimal distance  $dx$  to each other, and having  $dv_y$  difference in their velocities in the direction of  $y$  (that is orthogonal to the direction of  $x$ ), one can make the following statement:

$$\frac{F}{A} = -\eta \frac{dv_y}{dx}$$

that is commonly referred to as *Newton's law*. The  $\eta$  coefficient is called the viscosity of the fluid.

#### *Hagen-Poiseuille law*

The Ostwald viscometer works on the basis of the Hagen-Poiseuille law: the  $V$  volume of a homogeneous fluid having  $\eta$  viscosity passing per  $t$  time through a capillary tube having  $l$  length and  $r$  radius at  $p$  pressure can be written:

$$V = \frac{pr^4\pi}{8l\eta}t$$

The flow is laminar if the direction of the flow velocity is parallel in the parallel layers. The condition of the laminar flow is the low flow velocity and the radius of the pipe must be under a critical value. The critical value can be calculated from the Reynolds number ( $R_e = \rho v r / \eta$ , where  $\rho$  is the density,  $v$  is the velocity,  $r$  is the characteristic length in the system and  $\eta$  is the viscosity) which should be under 2100 in case of laminar flow. The pipe should be long enough and the out flow should go to a wider reservoir which already contains the fluid.

### The Ostwald viscometer

The Ostwald viscometer fulfils these conditions: a U-tube with two reservoir bulbs separated by a capillary as shown in Fig. 2.

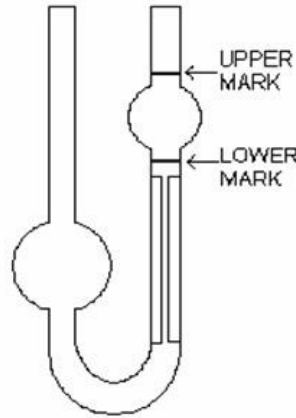


Fig. 2 Ostwald viscometer

The liquid is added to the viscometer, pulled into the upper reservoir by suction, and then allowed to drain by gravity back into the lower reservoir. The time that it takes for the liquid to pass between two etched marks, one above and one below the upper reservoir, is measured. If the level of the liquid having density  $\rho$  is initially at  $h_1$  and finally at  $h_2$  the mean hydrostatic pressure during the outflow is:

$$p = \frac{h_1 + h_2}{2} \rho g$$

where  $g$  is the gravitational acceleration of free fall. For absolute measurement we have to know all parameters of the viscometer ( $V, r, l, h_1, h_2$ ) but we can calibrate the equipment using a reference liquid having well known density  $\rho_0$ . The relative viscosity is:

$$\eta_{rel} = \frac{\eta}{\eta_0} = \frac{\rho t}{\rho_0 t_0}$$

where  $\rho$  is the density,  $t$  is the time of outflow of the sample,  $\rho_0$  is the density,  $t_0$  is the time of the outflow of the reference liquid (water). Knowing  $\eta_0$  the viscosity of the sample can be calculated.

We have to measure the density of the sample. At the laboratory course we use pycnometer for the determination of the density. We measure the weight of the empty pycnometer ( $m_p$ ) the filled pycnometer with reference liquid (water) ( $m_{p+w}$ ) having well known density ( $\rho_w$ ) and filled pycnometer with the sample ( $m_{p+s}$ ). The density of the sample ( $\rho_s$ ) can be calculated as follows:

$$\rho_s = \frac{m_{p+s} - m_p}{m_{p+w} - m_p} \rho_w$$

Task:

Determination of the relative and dynamic viscosity of the  $\text{CuSO}_4$  solution at  $20\text{ }^\circ\text{C}$ .

Equipments:

Ostwald viscometer, pycnometer,  $20\text{ cm}^3$  pipette, capillary funnel, thermostat, analytical balance

Steps:

1. Set up the temperature given by the supervisor of the measurement.
2. Measure  $20\text{ cm}^3$  distilled water to the lower reservoir and put the viscometer to the thermostat (even the  $h_1$  level should be under the level of the water in the thermostat)!
3. Measure the weight of the empty, dry(!) pycnometer!
4. Fill the pycnometer with 2x distilled water and put the pycnometer to the thermostat!
5. If the temperature is stable measure the time of the outflow 6-8 times! There should be no systematic change in the times!
6. Remove the distilled water from the viscometer and fill it with the  $\text{CuSO}_4$  solution! Put the viscometer to the thermostat again!
7. Fill the pycnometer to the ring mark IN the thermostat! Measure the weight of the pycnometer filled with 2x distilled water!
8. Clean the pycnometer, fill it with the  $\text{CuSO}_4$  solution and put to the thermostat again!
9. Measure the outflow times for the sample 6-8 times! There should be no systematic change in the times!
10. Fill the pycnometer to the ring mark IN the thermostat! Measure the weight of the pycnometer filled with the sample!
11. Clean the equipments!

Bp, 06/11/2014

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(The Viscous Flow part was written by Soma Vesztergom.)